Book Title: Elements of Geometry and Trigonometry

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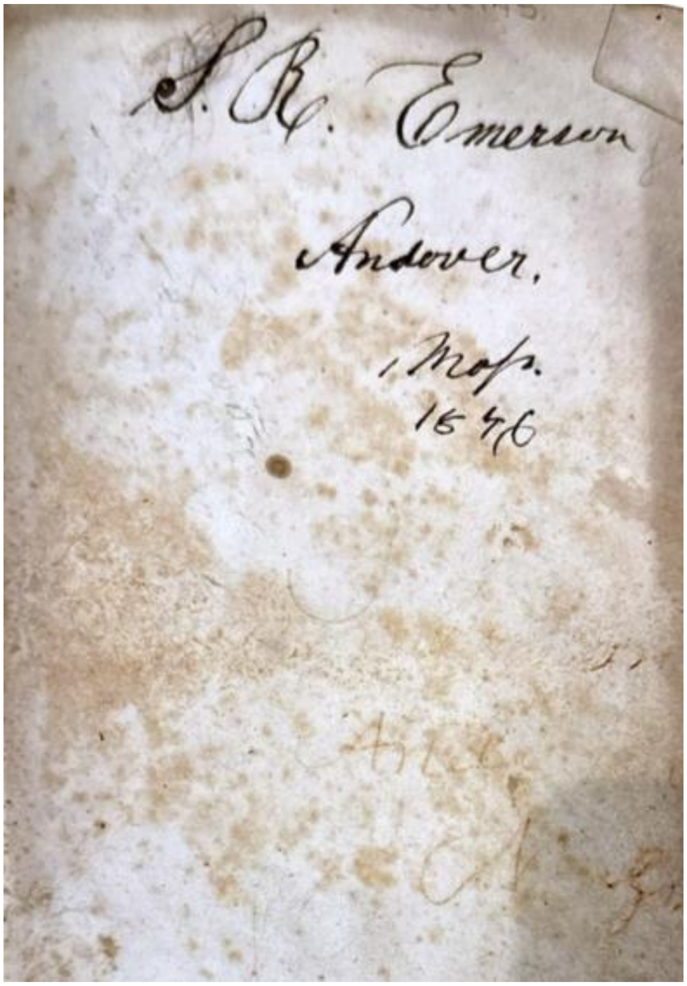
Page Number: Interior Cover ii



"S. R. Emerson
Andover,
Mass.
1846"

Further down there appears to be light brown letter markings similar to "Avieta N G"

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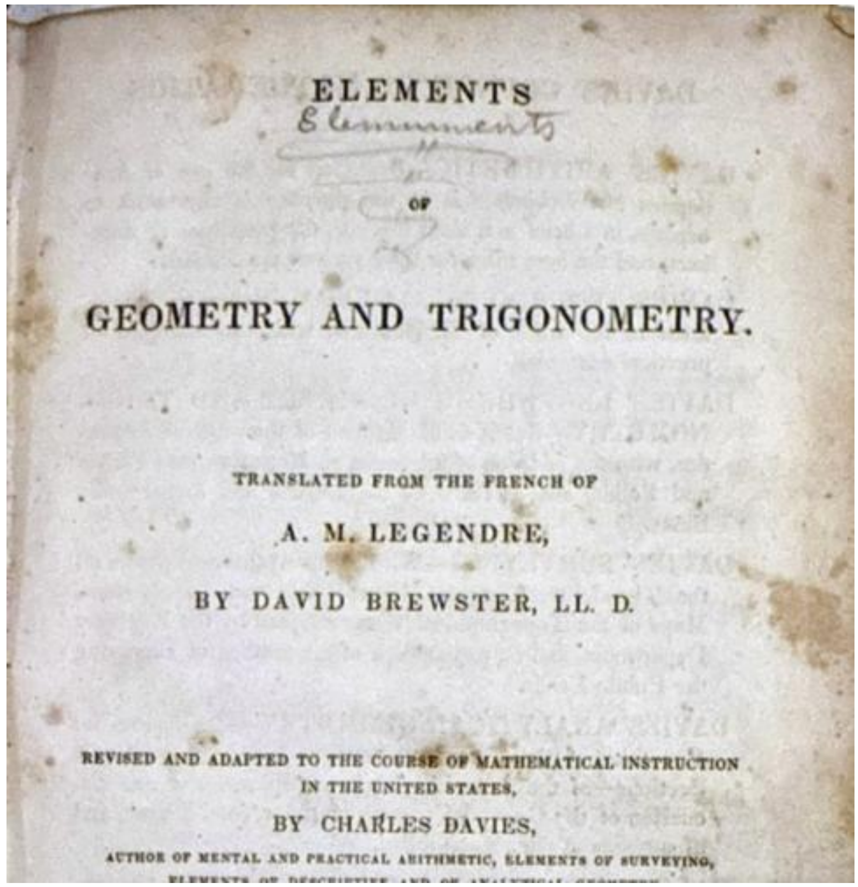
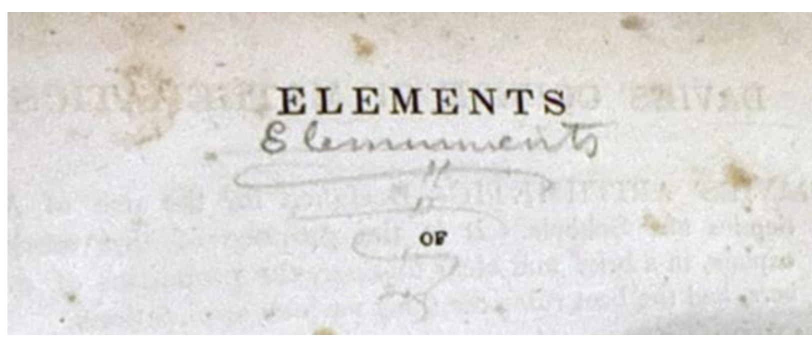


ELEMENTS
Elemmments
OF
GEOMETRY AND TRIGONOMETRY.
TRANSLATED FROM THE FRENCH OF
A. M. LEGENDRE,
BY DAVID BREWSTER, LL. D.
REVISED AND ADAPTED TO THE COURSE OF MATHEMATICAL INSTRUCTION IN THE UNITED STATES,
BY CHARLES DAVIES,
AUTHOR OF MENTAL AND PRACTICAL ARITHMETIC, ELEMENTS OF SURVEYING,



"Elemmments"

Fancy lines

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BOOK I.

THE PRINCIPLES.

Definitions.

1. GEOMETRY is the science which has for its object the measurement of extension.

Extension has three dimensions, length, breadth, and height, or thickness.

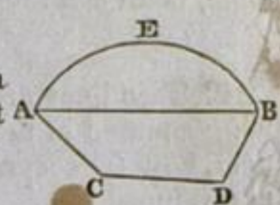
2. A *line* is length without breadth, or thickness.

The extremities of a line are called *points:* a point, therefore, has neither length, breadth, nor thickness, but position only.

3. A *straight line* is the shortest distance from one point to another.

4. Every line which is not straight, or composed of straight lines, is a *curved line.*

Thus, AB is a straight line; ACDB is a *broken* line, or one composed of straight lines; and AEB is a curved line.

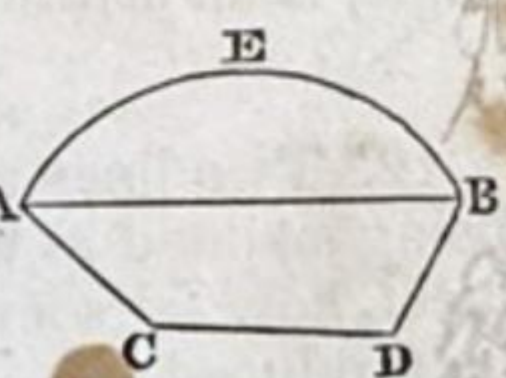


The word *line*, when used alone, will designate a straight line; and the word *curve*, a curved line.

5. A *surface* is that which has length and breadth, without

Underlining under "three"

"How do you prove this of EB, as of li "DB", is not a straight line"



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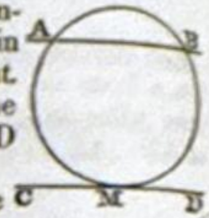
An *inscribed triangle* is one which, like BAC, has its three angular points in the circumference.

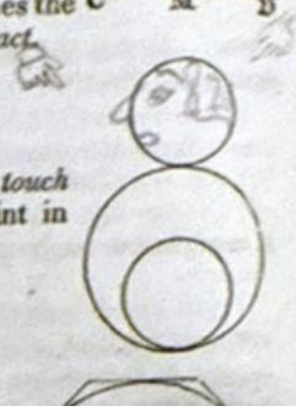
And, generally, an *inscribed figure* is one, of which all the angles have their vertices in the circumference. The circle is then said to *circumscribe* such a figure.

7. A *secant* is a line which meets the circumference in two points, and lies partly within and partly without the circle. AB is a secant.

8. A *tangent* is a line which has but one point in common with the circumference. CD is a tangent.



The point M, where the tangent touches the circumference, is called the *point of contact*.



In like manner, two circumferences *touch* each other when they have but one point in common.

Drawing of a man using illustrated circles, two hands pointing down to the drawing.

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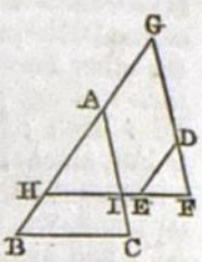
PROPOSITION XXI. THEOREM.

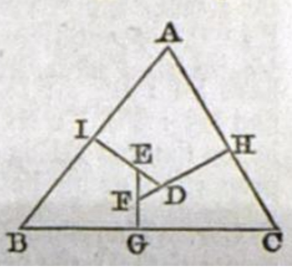
Two triangles, which have their homologous sides parallel, or perpendicular to each other, are similar.

Let BAC, EDF, be two triangles.

First. If the side AB is parallel to DE, and BC to EF, the angle ABC will be equal to DEF (Book I. Prop. XXIV.); if AC is parallel to DF, the angle ACB will be equal to DFE, and also BAC to EDF; hence the triangles ABC, DEF, are equiangular; consequently they are similar (Prop. XVIII.).



Secondly. If the side DE is perpendicular to AB, and the side DF to AC, the two angles I and H of the quadrilateral AIDH will be right angles; and since all the four angles are together equal to four right angles (Book I. Prop. XXVI. Cor. 1.), the remaining two IAH, IDH, will be together equal to two right



“2” sign near top right hand corner

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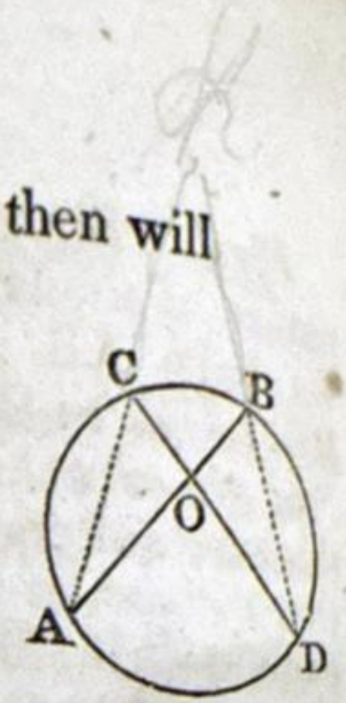
CD intersect at O : then will
)O : : OC : OB.
the triangles ACO,
equal, being verti-
to the angle D, be-
in the same segment
or. 1.) ; for the same
e triangles are there-
ologous sides give the proportion
DO : : CO : OB.



).OB=DO.CO: hence the rect...

Triangle extending beyond the top illustration with an "R" or some other letter indicating the angle.

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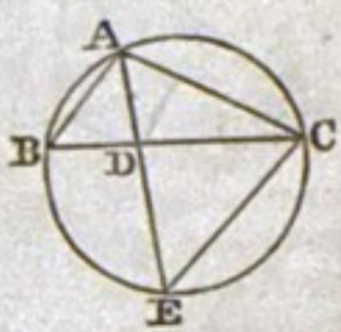
et AD bisect the angle A ; then will
C=AD²+BD.DC.
gh the three points
l it meets the cir-
E.
imilar to the trian-
othesis, the angle
angle B=E, since
by half of the arc
es are similar, and
e the proportion BA : AE : : AD :
.AD ; but AE=AD+DE, and multi-
als by AD, we have AE.AD=AD²+
=BD.DC (Prop. XXVIII.) ; hence,

C=AD²+BD.DC.



Quadrilateral drawing on top of illustration.

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BOOK V.

REGULAR POLYGONS, AND THE MEASUREMENT OF THE CIRCLE.

Definition.

A Polygon, which is at once equilateral and equiangular, is called a *regular polygon*.

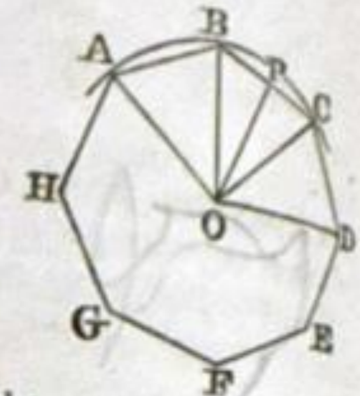
Regular polygons may have any number of sides: the equilateral triangle is one of three sides; the square is one of four.

Unknown writing to the right of the polygon definition.

PROPOSITION I. THEOREM.

Two regular polygons of the same number of sides are similar

Let ABCDE &c. be a regular polygon: describe a circle through the three points A, B, C, the centre being O, and OP the perpendicular let fall from it, to the middle point of BC: draw AO and OD.



If the quadrilateral OPCD be placed upon the quadrilateral OPBA, they will coincide; for the side OP is common; the angle OPC=OPB, each being a right angle; hence the side PC will apply to its equal PB, and the point C will fall on B: besides, from the nature of the polygon, the angle PCD=PBA; hence CD will take the direction BA; and since CD=BA, the point D will fall on A, and the two quadrilaterals will entirely coincide. The distance OD is therefore equal to AO; and consequently the circle which passes through the three points A, B, C, will also pass through the point D. By the same mode of reasoning, it might be shown, that the circle which passes through the three points B, C, D, will also pass through the point E; and so of all the rest: hence the circle which passes through the points A, B, C, passes also through the vertices of all the angles in the polygon, which is therefore inscribed in this circle.

Again, in reference to this circle, all the sides AB, BC, CD, &c. are equal chords; they are therefore equally distant from the centre (Book III. Prop. VIII.): hence, if from the point O with the distance OP, a circle be described, it will touch the side BC, and all the other sides of the polygon, each in its middle point, and the circle will be inscribed in the polygon, or the polygon described about the circle.

Scholium 1. The point O, the common centre of the inscribed and circumscribed circles, may also be regarded as the centre of the polygon; and upon this principle the angle AOB is called *the angle at the centre*, being formed by two radii drawn to the extremities of the same side AB.

Since all the chords AB, BC, CD, &c. are equal, all the angles at the centre must evidently be equal likewise; and therefore the value of each will be found by dividing four right angles by the number of sides of the pol

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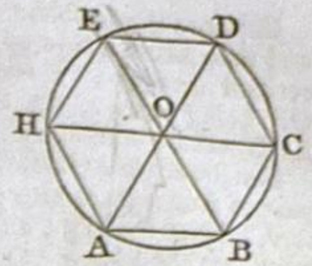
Scholium 2. To inscribe a regular polygon of a certain number of sides in a given circle, we have only to divide the circumference into as many equal parts as the polygon has sides: for the arcs being equal, the chords AB, BC, CD, &c. will also be equal; hence likewise the triangles AOB, BOC, COD, must

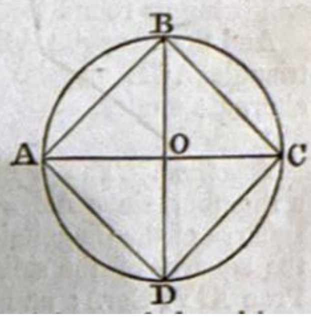


be equal, because the sides are equal each to each; hence all the angles ABC, BCD, CDE, &c. will be equal; hence the figure ABCDEH, will be a regular polygon.

PROPOSITION III. PROBLEM.

To inscribe a square in a given circle.

Draw two diameters AC, BD, cutting each other at right angles; join their extremities A, B, C, D: the figure ABCD will be a square. For the angles AOB, BOC, &c. being equal, the chords AB, BC, &c. are also equal: and the angles ABC, BCD, &c. being in semicircles, are right angles.



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scribed about the circle, we may prove in a similar way, the figure having the greatest number of sides will be the lea and the same may be shown, whatever be the number of sid of the polygons: hence, in general, *any circumscribed regul polygon, will be greater than a circumscribed regular polyg having double the number of sides.*

PROPOSITION VIII. THEOREM.

Two regular polygons, of the same number of sides, can alwa be formed, the one circumscribed about a circle, the other i scribed in it, which shall differ from each other by less tha any assignable surface.

Let Q be the side of a square less than

2 or scribble