Book Title: Elements of Geometry and Trigonometry

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and some have had patience enough to continue these decimals to the hundred and twenty-seventh, or even to the hundred and fortieth place. Such an approximation is evidently equivalent to perfect correctness: the root of an imperfect power is in no case more accurately known.

The following problem will exhibit one of the simplest elementary methods of obtaining those approximations.

PROPOSITION XIII. PROBLEM.

The surface of a regular inscribed polygon, and that of a similar polygon circumscribed, being given; to find the surfaces of the regular inscribed and circumscribed polygons having double the number of sides.

Let AB be a side of the given inscribed polygon; EF, parallel to AB, a side of the circumscribed

Scribble on right hand side.

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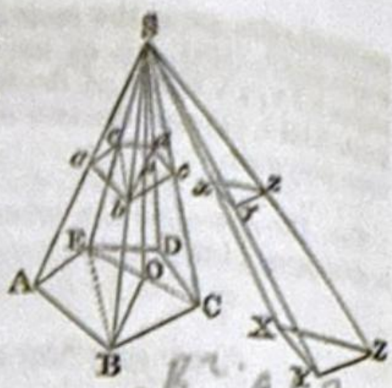
Cor. 1. Let S-ABCDE, S-XYZ be two pyramids, having a common vertex and the same altitude, or having their bases situated in the same plane; if these pyramids are cut by a plane parallel to the plane of their bases, giving the sections *abcde*, *xyz*, then will the sections *abcde*, *xyz*, be to each other as the bases ABCDE, XYZ.



For, the polygons ABCDE, *abcde*, being similar, their surfaces are as the squares of the homologous sides AB, *ab*; but AB : *ab* : : SA : S*a*; hence ABCDE : *abcde* : : SA^2 : Sa^2. For the same reason, XYZ : *xyz* : : SX^2 : Sx^2. But since *abc* and *xyz* are in one plane, we have likewise SA : S*a* : : SX : S*x* (Book VI. Prop. XV.); hence ABCDE : *abcde* : : XYZ : *xyz*; hence the sections *abcde*, *xyz*, are to each other as the bases ABCDE, XYZ.

"ab²:ab²"

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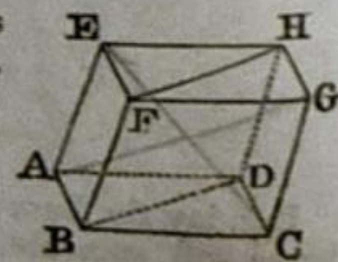
PROPOSITION VI. THEOREM.

In every parallelopipedon the opposite planes are equal and parallel.

By the definition of this solid, the bases ABCD, EFGH, are equal parallelograms, and their sides are parallel: it remains only to show, that the same is true of any two opposite lateral faces, such as AEHD, BFGC. Now AD is equal and parallel to BC, because the figure ABCD is a par-



"Mon."

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PROPOSITION XI. THEOREM.

Two rectangular parallelopipedons, which have the same base, are to each other as their altitudes.

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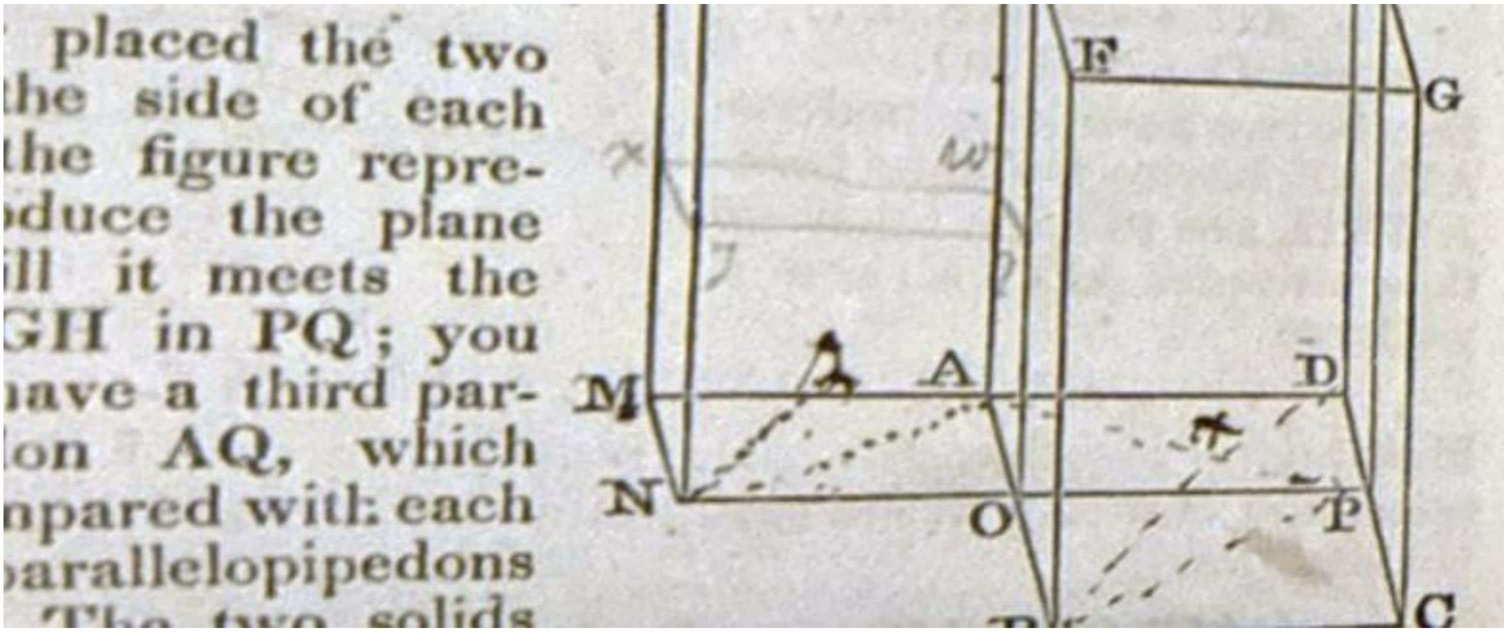
placed the two
the side of each
the figure repre-
oduce the plane
ill it meets the
GH in PQ; you
have a third par-
on AQ, which
npared with each
parallelopipedons

Drawing extending the rectangular parallelopipedons with drawn letters and dash marks.

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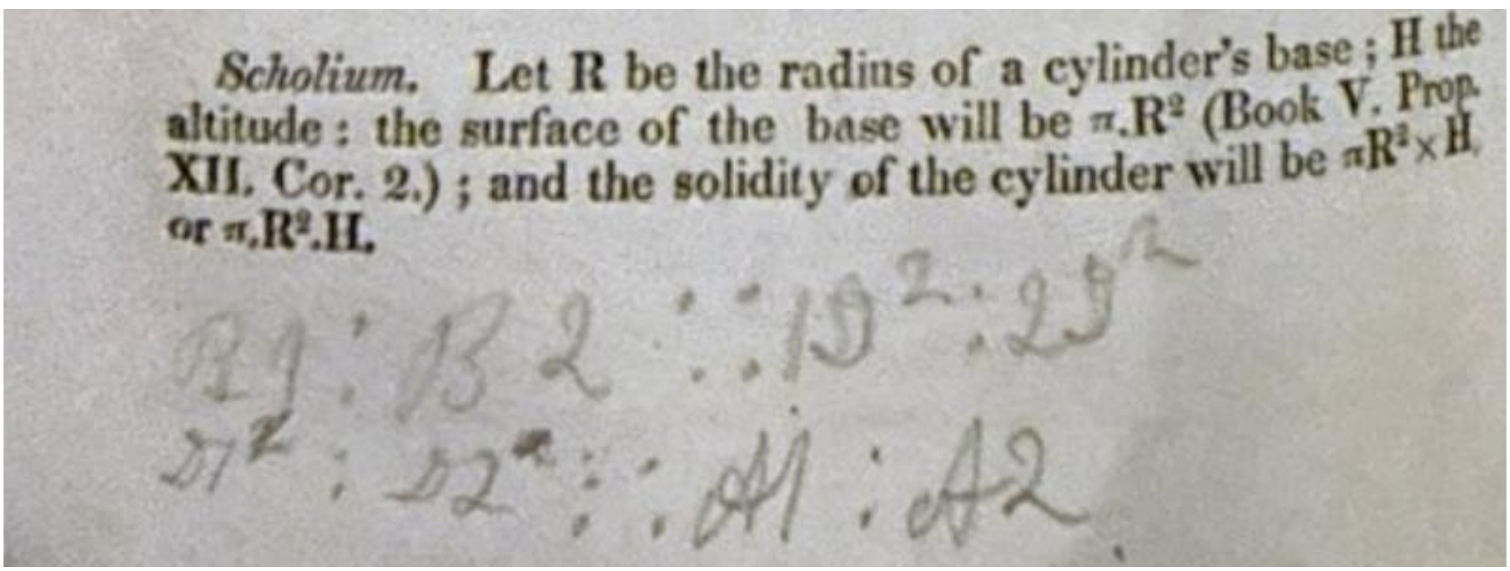
Scholium. Let R be the radius of a cylinder's base; H the altitude: the surface of the base will be $\pi.R^2$ (Book V. Prop. XII. Cor. 2.); and the solidity of the cylinder will be $\pi R^2 \times H$, or $\pi.R^2.H$.

"B1:B2::1D2:2D2"

"D12:D22::A1:A2"

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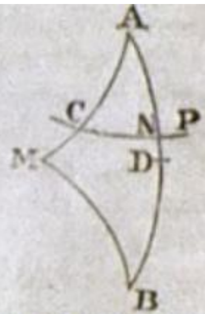

Let ANB be the arc of a great circle which joins the points A and B; then will it be the shortest path between them.

1st. If two points N and B, be taken on the arc of a great circle, at unequal distances from the point A, the shortest distance from B to A will be greater than the shortest distance from N to A.

For, about A as a pole describe a circumference CNP. Now, the line of shortest distance from B to A must cross this circumference at some point as P. But the shortest distance from P to A whether it be the arc of a great circle or any other line, is equal to the shortest distance from N to A; for, by passing the arc of a great circle through P and A, and revolving it about the

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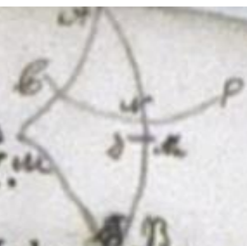

est distance possible; then through M draw MA, MB, ar great circles, and take BD equal to BM. By the last the BDA < BM + MA; take BD = BM from each, and there w main AD < AM. Now, since BM = BD, the shortest path fr to M is equal to the shortest path from B to D: hence if w pose two paths from B to A, one passing through M and the through D, they will have an equal part in each; viz. th from B to M equal to the part from B to D.

But by hypothesis, the path through M is the shortest path B to A: hence the shortest path from M to A must be less the shortest path from D to A, whereas it is greater sinc arc MA is greater than DA: hence, no point of the sh distance between B and A can lie out of the arc of the circle BDA.

PROPOSITION III. THEOREM.

The sum of the three sides of a spherical triangle is less th circumference of a great circle.

Let ABC be any spherical trian- C

Two drawings of mathematical geometrical shapes.

"I must nonsensical denouth dealith???"

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PLANE TR

The *secant* of an arc is th
the circle through one extrem
tangent drawn through the ot
secant of the arc AM, or of tl
The *versed sine* of an arc, i
cepted between one extremity
sine. Thus, AP is the versed s
ACM.
These four lines MP, AT, C
arc AM, and are always detern
are thus designated :
MP=sin AM

"R"

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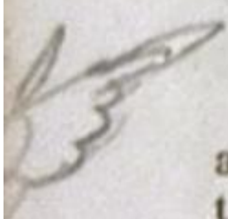
the circle, we have

$$\sin 0=0, \ \ \text{tang}\, 0=0, \ \ \cos 0=R, \ \ \sec 0=R.$$

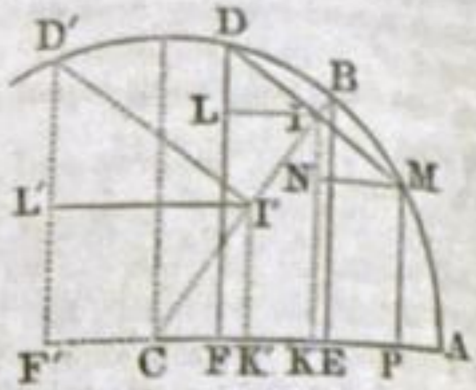


VIII. As the point M advances towards D, the sine increases, and so likewise does the tangent and the secant; but the cosine, the cotangent, and the cosecant, diminish.

When the point M is at the middle of AD, or when the arc AM is 45°, in which case it is equal to its complement MD,

Drawing of a hand on bottom left side.

XIX. *The sines and cosines of two arcs, a and b, being given, it is required to find the sine and cosine of the sum or difference of these arcs.*



Let the radius AC=R, the arc AB=a, the arc BD=b, and consequently ABD=$a+b$. From the points B and D, let fall the perpendiculars BE, DF upon AC; from the point D, draw DI perpendicular to BC; lastly, from the point I draw IK perpendicular, and IL parallel to, AC.

The similar triangles BCE, ICK, give the proportions,

CB : CI : : BE : IK, or R : cos b : : sin a : IK=$\frac{\sin a \cos b}{R}$.

CB : CI : : CE : CK, or R : cos b : : cos a : CK=$\frac{\cos a \cos b}{R}$.

The triangles DIL, CBE, having their sides perpendicular, each to each, are similar, and give the proportions,

CB : DI : : CE : DL, or R : sin b : : cos a : DL=$\frac{\cos a \sin b}{R}$.

CB : DI : : BE : IL, or R : sin b : : sin a : IL=$\frac{\sin a \sin b}{R}$.

But we have

IK+DL=DF=sin $(a+b)$, and CK—IL=CF=cos $(a+b)$.

Hence

$$\sin(a+b)=\frac{\sin a \cos b+\sin b \cos a}{R}$$

$$\cos(a+b)=\frac{\cos a \cos b-\sin a \sin b}{R}.$$

The values of sin $(a-b)$ and of cos $(a-b)$ might be easily deduced from these two formulas; but they may be found

"a" over geometrical figure

"you de seamh"

"you C.S."

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If we put $2a$ in the place of a, we shall have,

$$\sin a = \sqrt{(\tfrac{1}{2}R^2 - \tfrac{1}{2}R\cos 2a)} = \tfrac{1}{2}\sqrt{2R^2 - 2R\cos 2a}.$$

$$\cos a = \sqrt{(\tfrac{1}{2}R^2 + \tfrac{1}{2}R\cos 2a)} = \tfrac{1}{2}\sqrt{2R^2 + 2R\cos 2a}.$$

Making, in the two last formulas, $a = 45°$, gives $\cos 2a = 0$, and

$\sin 45° = \sqrt{\tfrac{1}{2}R^2} = R\sqrt{\tfrac{1}{2}}$; and also, $\cos 45° = \sqrt{\tfrac{1}{2}R^2} = R\sqrt{\tfrac{1}{2}}$.

Next, make $a = 22° 30'$, which gives $\cos 2a = R\sqrt{\tfrac{1}{2}}$, and we have

$\sin 22° 30' = R\sqrt{(\tfrac{1}{2} - \tfrac{1}{2}\sqrt{\tfrac{1}{2}})}$ and $\cos 22° 30' = R\sqrt{(\tfrac{1}{2} + \tfrac{1}{2}\sqrt{\tfrac{1}{2}})}$.

XXI. If we multiply together formulas (1.) and (2.) Art. XIX. and substitute for $\cos^2 a$, $R^2 - \sin^2 a$, and for $\cos^2 b$, $R^2 - \sin^2 b$; we shall obtain, after reducing and dividing by R^2,

$$\sin(a+b)\sin(a-b) = \sin^2 a - \sin^2 b = (\sin a + \sin b)(\sin a - \sin b).$$

or, $\sin(a-b) : \sin a - \sin b :: \sin a + \sin b : \sin(a+b)$.

XXII. The formulas of Art. XIX. furnish a great number of consequences; among which it will be enough to mention those of most frequent use. By adding and subtracting we obtain the four which follow,

$$\sin(a+b) + \sin(a-b) = \frac{2}{R}\sin a\cos b.$$

$$\sin(a+b) - \sin(a-b) = \frac{2}{R}\sin b\cos a.$$

$$\cos(a+b) + \cos(a-b) = \frac{2}{R}\cos a\cos b.$$

$$\cos(a-b) - \cos(a+b) = \frac{2}{R}\sin a\sin b.$$

and which serve to change a product of several sines or cosines into *linear* sines or cosines, that is, into sines and cosines multiplied only by constant quantities.

XXIII. If in these formulas we put $a+b=p$, $a-b=q$, which gives $a = \frac{p+q}{2}$, $b = \frac{p-q}{2}$, we shall find

$$\sin p + \sin q = \frac{2}{R}\sin\tfrac{1}{2}(p+q)\cos\tfrac{1}{2}(p-q) \quad (1.)$$

$$\sin p - \sin q = \frac{2}{R}\sin\tfrac{1}{2}(p-q)\cos\tfrac{1}{2}(p+q) \quad (2.)$$

$$\cos p + \cos q = \frac{2}{R}\cos\tfrac{1}{2}(p+q)\cos\tfrac{1}{2}(p-q) \quad (3.)$$

$$\cos q - \cos p = \frac{2}{R}\sin\tfrac{1}{2}(p+q)\sin\tfrac{1}{2}(p-q) \quad (4.)$$

"2R times 0 = 0"
"2a = 45 degrees"
"a.b/#"
"xsin2"
"you hasir"