Book Title: Elements of Geometry and Trigonometry

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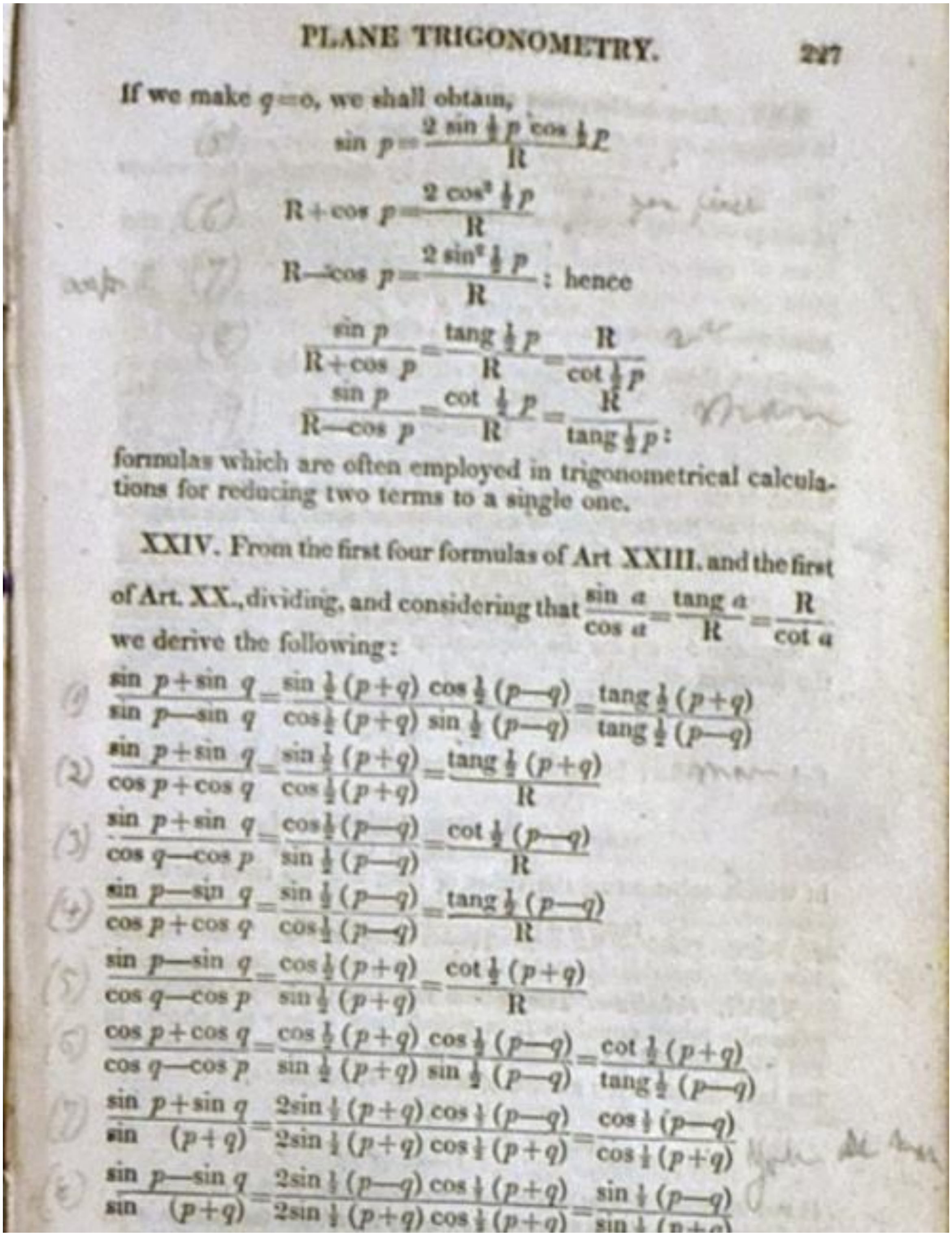

If we make $q=o$, we shall obtain,

(5) $$\sin p=\frac{2\sin\frac{1}{2}p\cos\frac{1}{2}p}{R}$$

(6) $$R+\cos p=\frac{2\cos^2\frac{1}{2}p}{R}$$

(7) $$R-\cos p=\frac{2\sin^2\frac{1}{2}p}{R}: \text{ hence}$$

(8) $$\frac{\sin p}{R+\cos p}=\frac{\tan\mathrm{g}\,\frac{1}{2}p}{R}=\frac{R}{\cot\frac{1}{2}p}$$

(9) $$\frac{\sin p}{R-\cos p}=\frac{\cot\frac{1}{2}p}{R}=\frac{R}{\tan\mathrm{g}\,\frac{1}{2}p}:$$

formulas which are often employed in trigonometrical calculations for reducing two terms to a single one.

XXIV. From the first four formulas of Art XXIII. and the first of Art. XX., dividing, and considering that $\frac{\sin a}{\cos a}=\frac{\tan\mathrm{g}\,a}{R}=\frac{R}{\cot a}$ we derive the following:

(1) $$\frac{\sin p+\sin q}{\sin p-\sin q}=\frac{\sin\frac{1}{2}(p+q)\cos\frac{1}{2}(p-q)}{\cos\frac{1}{2}(p+q)\sin\frac{1}{2}(p-q)}=\frac{\tan\mathrm{g}\,\frac{1}{2}(p+q)}{\tan\mathrm{g}\,\frac{1}{2}(p-q)}$$

(2) $$\frac{\sin p+\sin q}{\cos p+\cos q}=\frac{\sin\frac{1}{2}(p+q)}{\cos\frac{1}{2}(p+q)}=\frac{\tan\mathrm{g}\,\frac{1}{2}(p+q)}{R}$$

(3) $$\frac{\sin p+\sin q}{\cos q-\cos p}=\frac{\cos\frac{1}{2}(p-q)}{\sin\frac{1}{2}(p-q)}=\frac{\cot\frac{1}{2}(p-q)}{R}$$

(4) $$\frac{\sin p-\sin q}{\cos p+\cos q}=\frac{\sin\frac{1}{2}(p-q)}{\cos\frac{1}{2}(p-q)}=\frac{\tan\mathrm{g}\,\frac{1}{2}(p-q)}{R}$$

(5) $$\frac{\sin p-\sin q}{\cos q-\cos p}=\frac{\cos\frac{1}{2}(p+q)}{\sin\frac{1}{2}(p+q)}=\frac{\cot\frac{1}{2}(p+q)}{R}$$

(6) $$\frac{\cos p+\cos q}{\cos q-\cos p}=\frac{\cos\frac{1}{2}(p+q)\cos\frac{1}{2}(p-q)}{\sin\frac{1}{2}(p+q)\sin\frac{1}{2}(p-q)}=\frac{\cot\frac{1}{2}(p+q)}{\tan\mathrm{g}\,\frac{1}{2}(p-q)}$$

(7) $$\frac{\sin p+\sin q}{\sin\ (p+q)}=\frac{2\sin\frac{1}{2}(p+q)\cos\frac{1}{2}(p-q)}{2\sin\frac{1}{2}(p+q)\cos\frac{1}{2}(p+q)}=\frac{\cos\frac{1}{2}(p-q)}{\cos\frac{1}{2}(p+q)}$$

(8) $$\frac{\sin p-\sin q}{\sin\ (p+q)}=\frac{2\sin\frac{1}{2}(p-q)\cos\frac{1}{2}(p+q)}{2\sin\frac{1}{2}(p+q)\cos\frac{1}{2}(p+q)}=\frac{\sin\frac{1}{2}(p-q)}{\sin\frac{1}{2}(p+q)}$$

"(5) (6) (7) (8) (9)"

"ooop"

"you find"

"22"

"Mon"

"(1) (2) (3) (4) (5) (6) (7) (8)"

"you do Mon"

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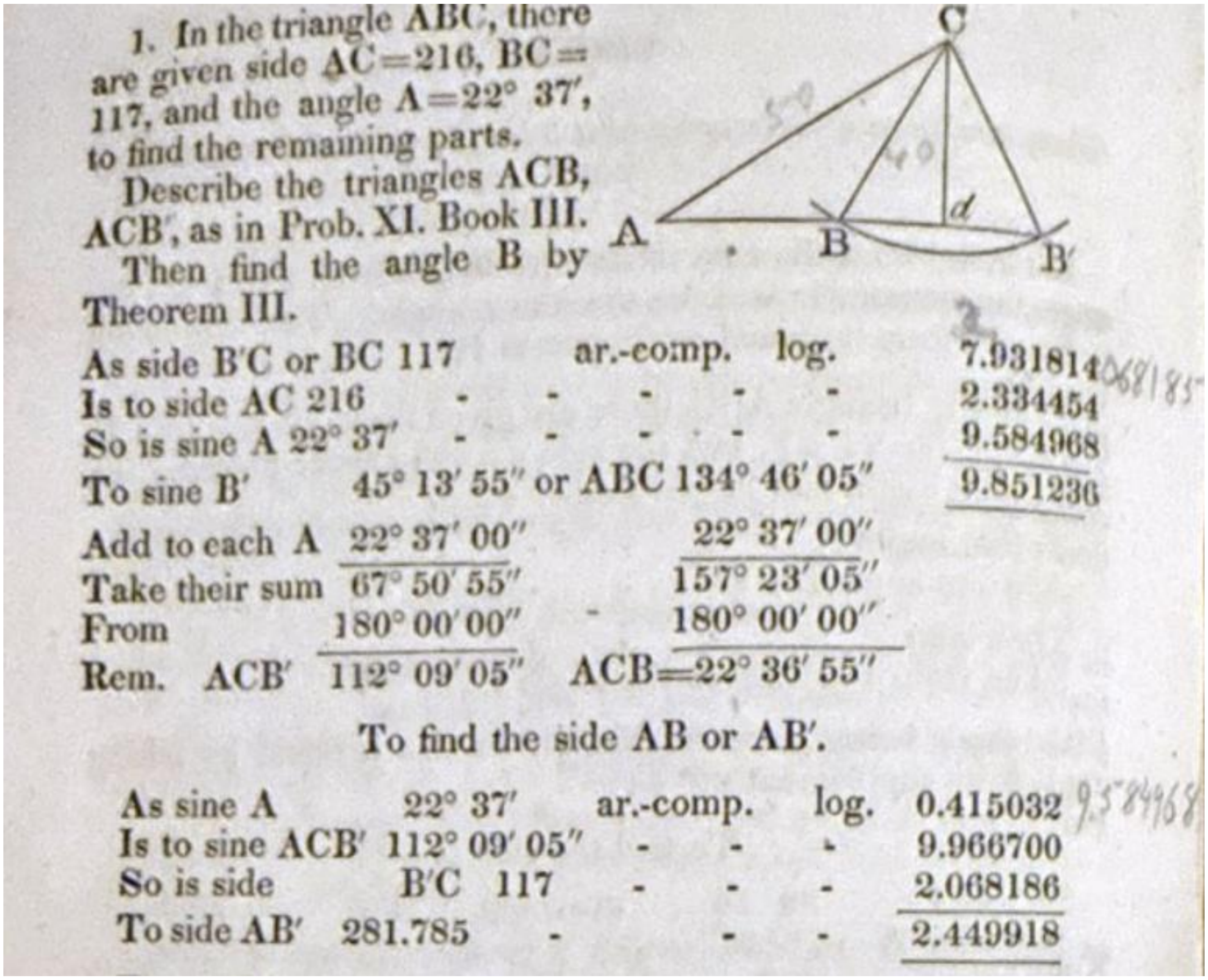

1. In the triangle ABC, there are given side AC=216, BC=117, and the angle A=22° 37′, to find the remaining parts.

Describe the triangles ACB, ACB′, as in Prob. XI. Book III.

Then find the angle B by Theorem III.

As side B′C or BC 117	ar.-comp. log.		2 7.931814 068185
Is to side AC 216	- - - - -		2.334454
So is sine A 22° 37′	- - - - - -		9.584968
To sine B′	45° 13′ 55″ or ABC 134° 46′ 05″		9.851236
Add to each A	22° 37′ 00″	22° 37′ 00″	
Take their sum	67° 50′ 55″	157° 23′ 05″	
From	180° 00′ 00″	180° 00′ 00″	
Rem. ACB′	112° 09′ 05″	ACB=22° 36′ 55″	

To find the side AB or AB′.

As sine A	22° 37′	ar.-comp. log.	0.415032 9.584968
Is to sine ACB′	112° 09′ 05″	- - -	9.966700
So is side	B′C 117	- - -	2.068186
To side AB′	281.785	- - -	2.449918

“50” “40” in triangle ABC

“ “068185” ”

“9.584968”

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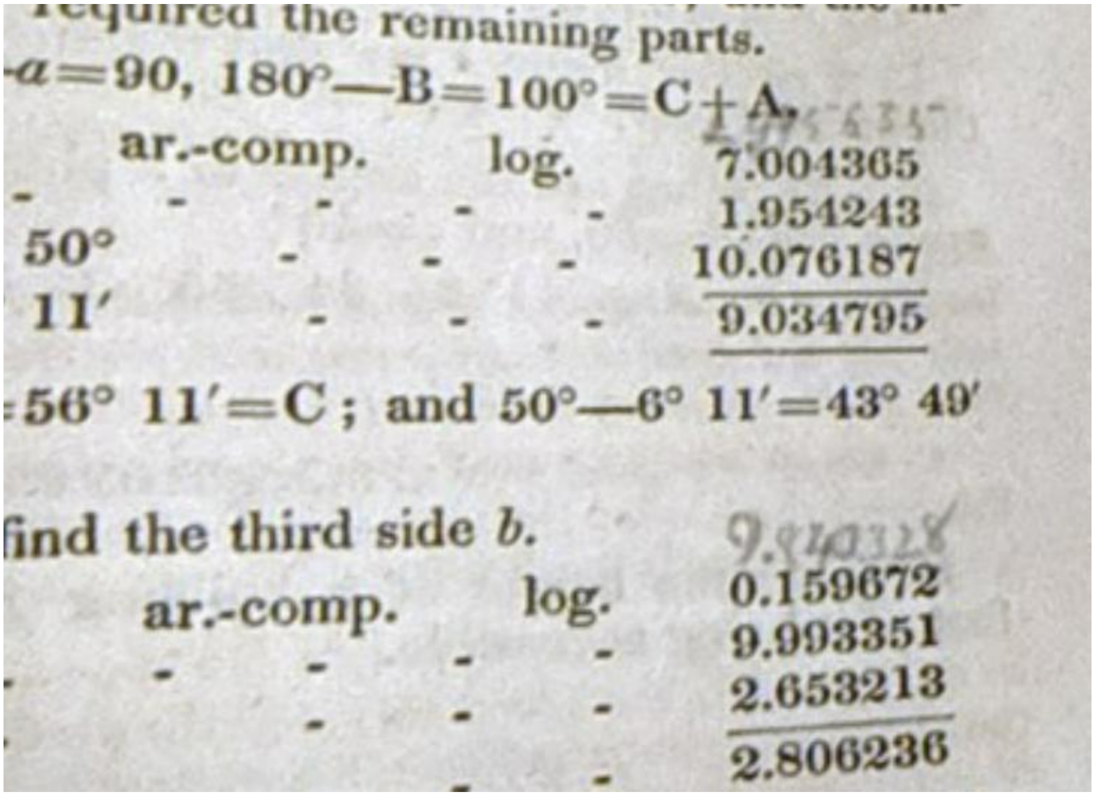

required the remaining parts.

-a=90, 180°—B=100°=C+A.

ar.-comp.	log.	2.495635 7.004365
- - -	- -	1.954243
50°	- - -	10.076187
11′	- - -	9.034795

=56° 11′=C; and 50°—6° 11′=43° 49′

find the third side b.

ar.-comp.	log.	9.840328 0.159672
- -	- -	9.993351
-	- -	2.653213
-	- -	2.806236

“2.495635”

“9.840328”

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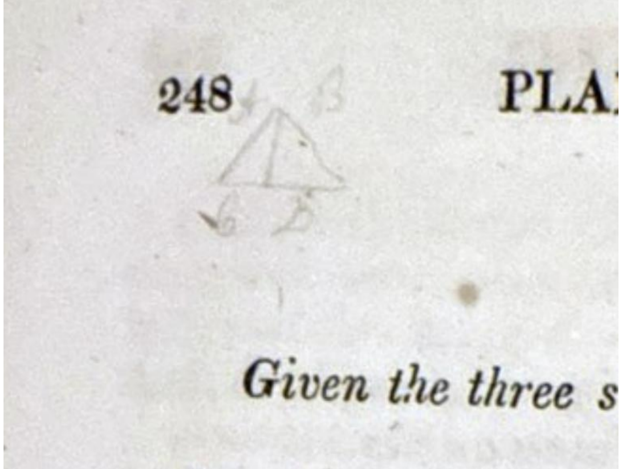

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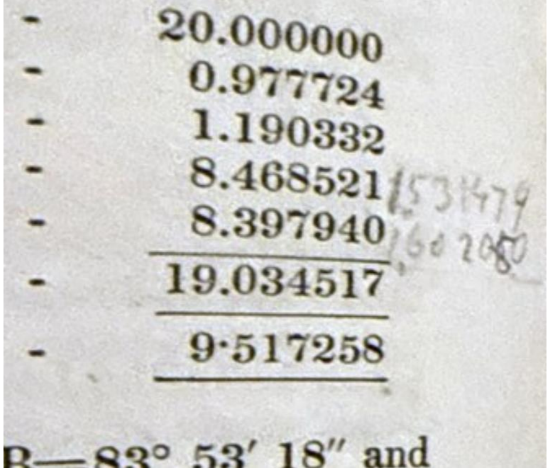
PLA

Given the three s

Drawing of a triangle with angles "A", "B", "C", "D"

"1.531479"

"1.602080"



-	20.000000
-	0.977724
-	1.190332
-	8.468521 1531479
-	8.397940 1602080
-	19.034517
-	9·517258

B—83° 53′ 18″ and

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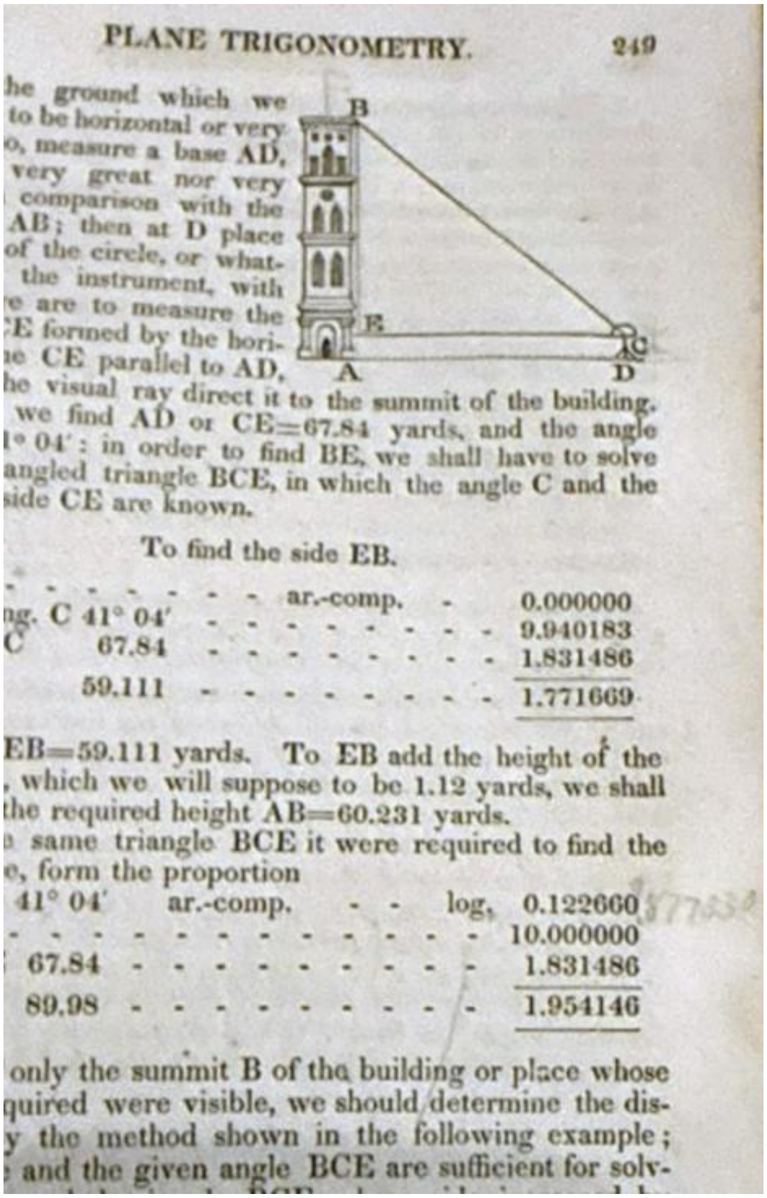


PLANE TRIGONOMETRY. 249

he ground which we
to be horizontal or very
o, measure a base AD,
very great nor very
comparison with the
AB; then at D place
of the circle, or what-
the instrument, with
e are to measure the
E formed by the hori-
ie CE parallel to AD,
he visual ray direct it to the summit of the building.
we find AD or CE=67.84 yards, and the angle
1° 04′: in order to find BE, we shall have to solve
angled triangle BCE, in which the angle C and the
side CE are known.

To find the side EB.

	- - - - - - -	ar.-comp. -	0.000000
ng. C 41° 04′	- - - - - - - - - -		9.940183
C 67.84	- - - - - - - - - -		1.831486
59.111	- - - - - - - - - -		1.771669

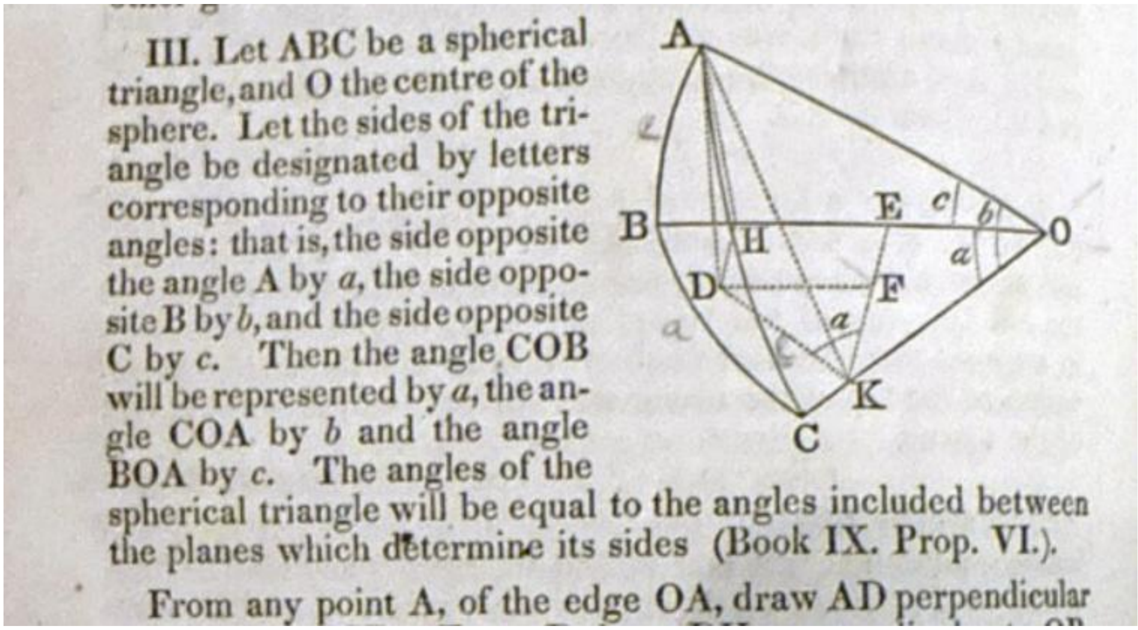
EB=59.111 yards. To EB add the height of the
, which we will suppose to be 1.12 yards, we shall
the required height AB=60.231 yards.
same triangle BCE it were required to find the
e, form the proportion

41° 04′	ar.-comp. - -	log,	0.122660
- - - - - - - - - - - - - -			10.000000
67.84	- - - - - - - - - - -		1.831486
89.98	- - - - - - - - - - -		1.954146

only the summit B of tha building or place whose
quired were visible, we should determine the dis-
y the method shown in the following example;
and the given angle BCE are sufficient for solv-

Drawn flag on top of building.

"9.877330"



III. Let ABC be a spherical triangle, and O the centre of the sphere. Let the sides of the triangle be designated by letters corresponding to their opposite angles: that is, the side opposite the angle A by *a*, the side opposite B by *b*, and the side opposite C by *c*. Then the angle COB will be represented by *a*, the angle COA by *b* and the angle BOA by *c*. The angles of the spherical triangle will be equal to the angles included between the planes which determine its sides (Book IX. Prop. VI.).

From any point A, of the edge OA, draw AD perpendicular

Letters drawn within the spherical figure.

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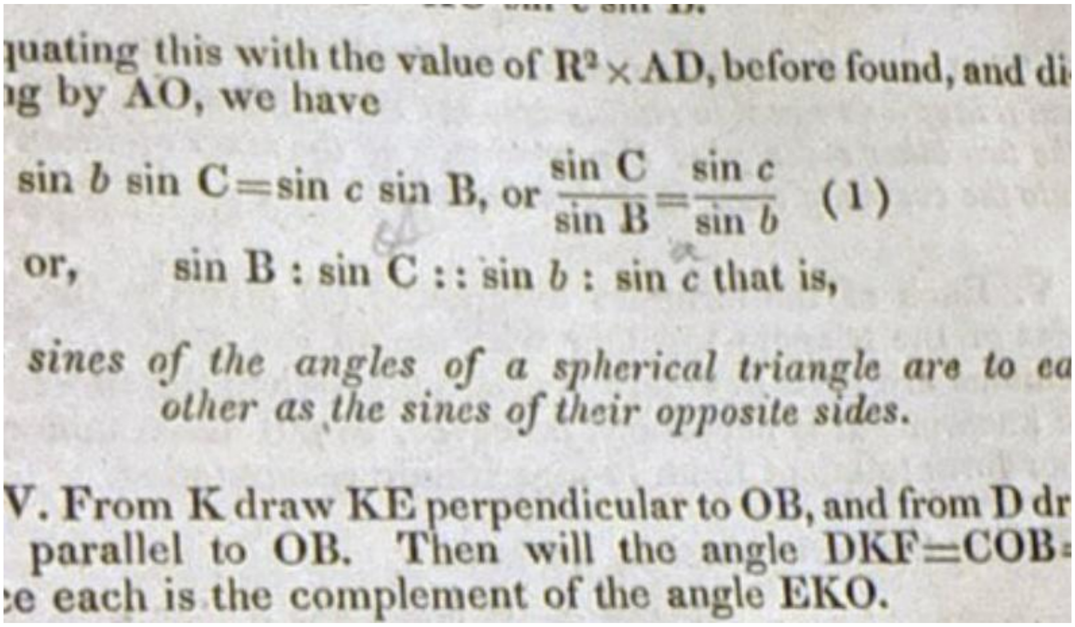


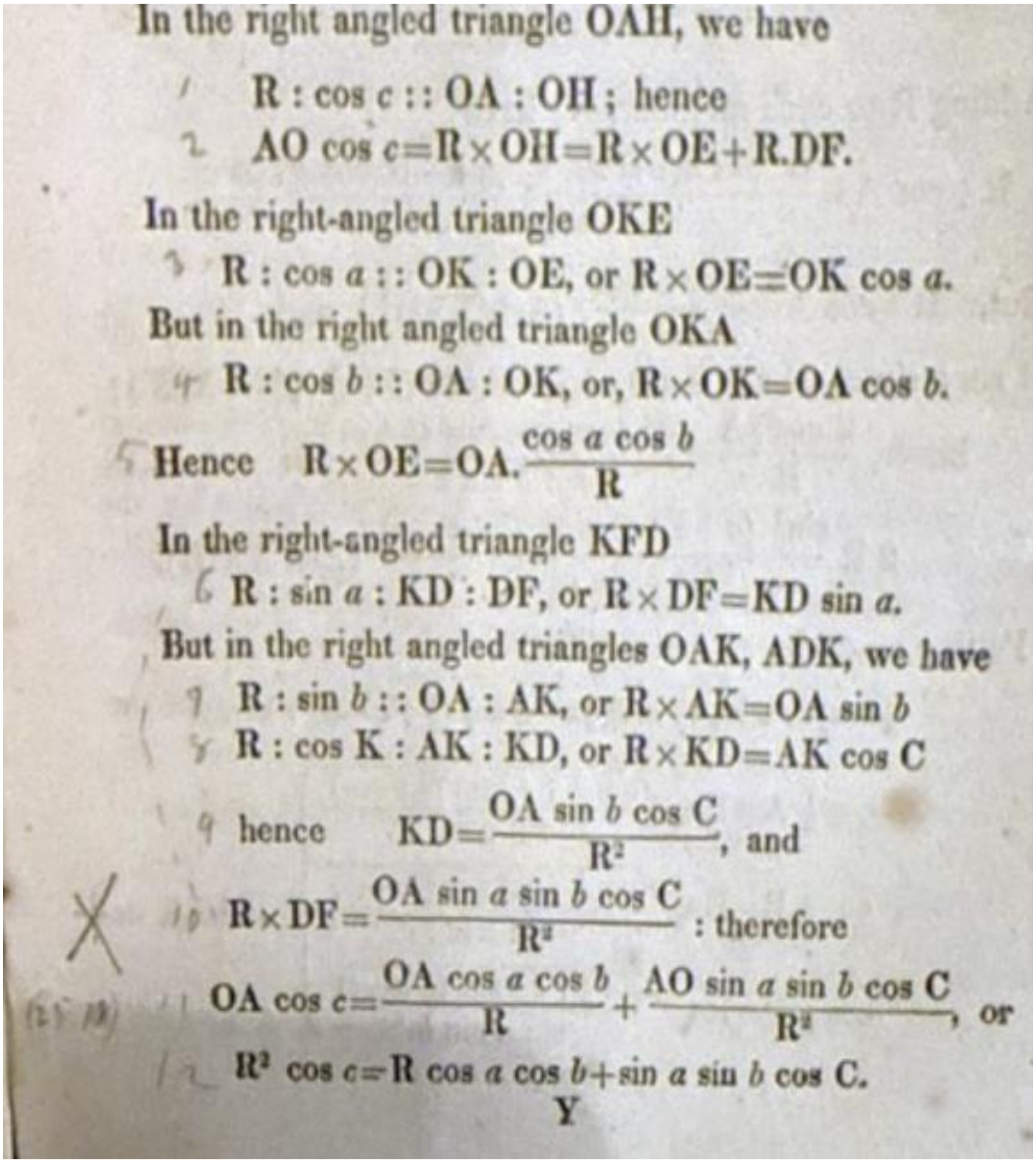
quating this with the value of $R^2 \times AD$, before found, and di
ng by AO, we have

$$\sin b \sin C = \sin c \sin B, \text{ or } \frac{\sin C}{\sin B} = \frac{\sin c}{\sin b} \quad (1)$$

or, $\sin B : \sin C :: \sin b : \overset{a}{\sin c}$ that is,

sines of the angles of a spherical triangle are to ea
other as the sines of their opposite sides.

V. From K draw KE perpendicular to OB, and from D dr
parallel to OB. Then will the angle DKF=COB
e each is the complement of the angle EKO.



In the right angled triangle OAH, we have

1 $R : \cos c :: OA : OH$; hence

2 $AO \cos c = R \times OH = R \times OE + R.DF.$

In the right-angled triangle OKE

3 $R : \cos a :: OK : OE$, or $R \times OE = OK \cos a$.

But in the right angled triangle OKA

4 $R : \cos b :: OA : OK$, or, $R \times OK = OA \cos b$.

5 Hence $R \times OE = OA. \dfrac{\cos a \cos b}{R}$

In the right-angled triangle KFD

6 $R : \sin a : KD : DF$, or $R \times DF = KD \sin a$.

But in the right angled triangles OAK, ADK, we have

7 $R : \sin b :: OA : AK$, or $R \times AK = OA \sin b$

8 $R : \cos K : AK : KD$, or $R \times KD = AK \cos C$

9 hence $KD = \dfrac{OA \sin b \cos C}{R^2}$, and

X 10 $R \times DF = \dfrac{OA \sin a \sin b \cos C}{R^2}$: therefore

(25 10) 11 $OA \cos c = \dfrac{OA \cos a \cos b}{R} + \dfrac{AO \sin a \sin b \cos C}{R^2}$, or

12 $R^2 \cos c = R \cos a \cos b + \sin a \sin b \cos C.$

Y

Marks above the sin C and sin C formulas.

“a” located over second sin C formula.

“1”
“2”
“3”
“4”
“5”
“6”
“7”
“8”
“9”
“10”
“11”
“12”

“x”
“(25 10)”

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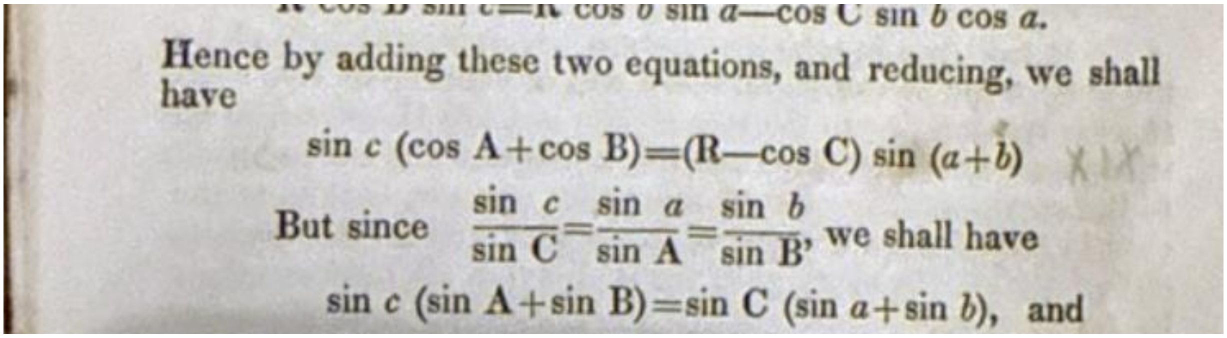

$$R \cos B \sin c = R \cos b \sin a - \cos C \sin b \cos a.$$

Hence by adding these two equations, and reducing, we shall have

$$\sin c\,(\cos A+\cos B)=(R-\cos C)\sin(a+b)$$ XIX

But since $\frac{\sin c}{\sin C}=\frac{\sin a}{\sin A}=\frac{\sin b}{\sin B}$, we shall have

$$\sin c\,(\sin A+\sin B)=\sin C\,(\sin a+\sin b),$$ and

Mark above the (a+b) and "XIX" next to the formula on right hand side.

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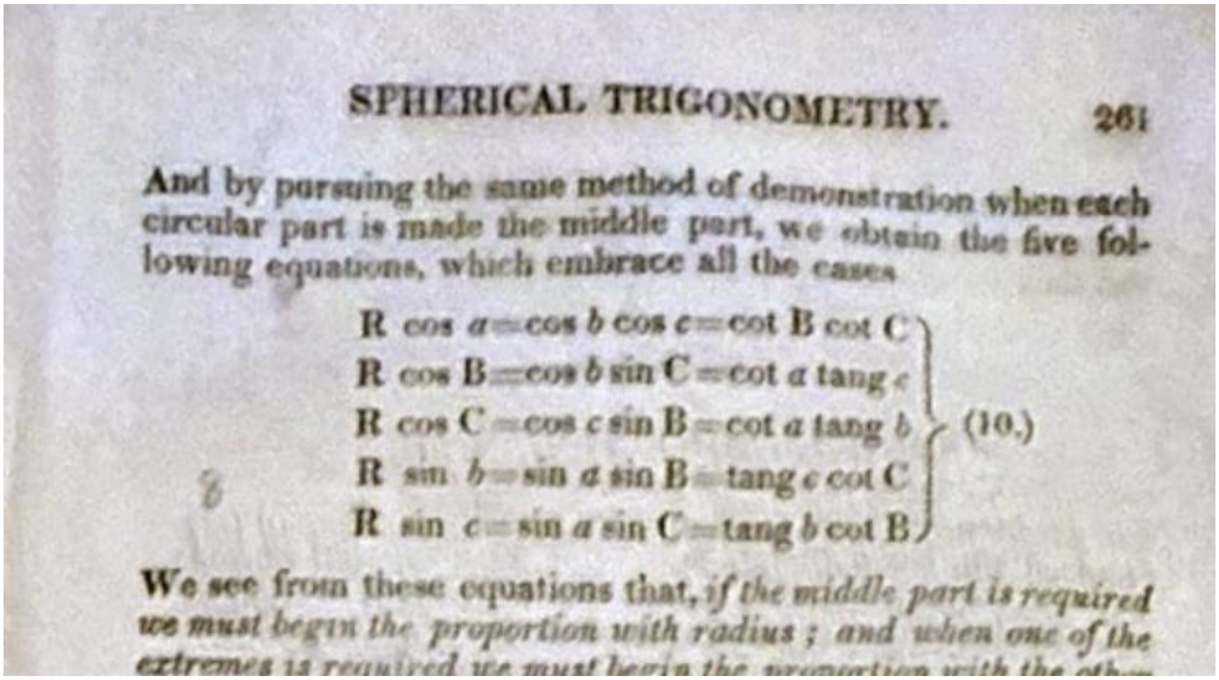

SPHERICAL TRIGONOMETRY. 261

And by pursuing the same method of demonstration when each circular part is made the middle part, we obtain the five following equations, which embrace all the cases

$$\left.\begin{array}{l} R\cos a=\cos b\cos c=\cot B\cot C \\ R\cos B=\cos b\sin C=\cot a\,\text{tang}\,c \\ R\cos C=\cos c\sin B=\cot a\,\text{tang}\,b \\ R\sin b=\sin a\sin B=\text{tang}\,c\cot C \\ R\sin c=\sin a\sin C=\text{tang}\,b\cot B \end{array}\right\}(10.)$$

8

We see from these equations that, *if the middle part is required we must begin the proportion with radius; and when one of the extremes is required we must begin the proportion with the other*

"8"

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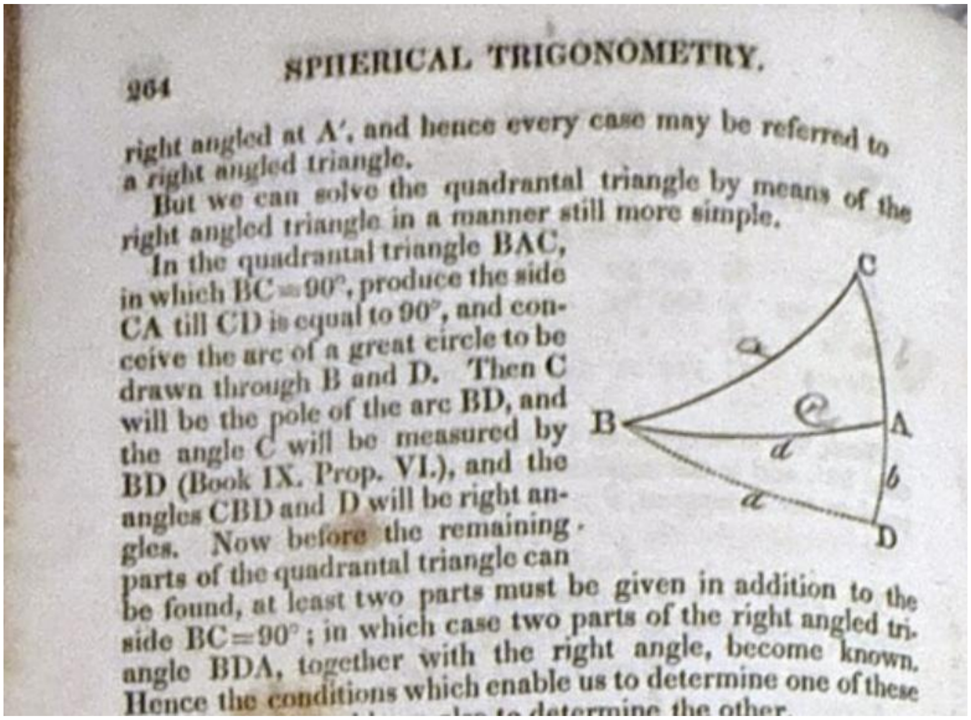

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right angled at A′, and hence every case may be referred to a right angled triangle.

But we can solve the quadrantal triangle by means of the right angled triangle in a manner still more simple.

In the quadrantal triangle BAC, in which BC=90°, produce the side CA till CD is equal to 90°, and conceive the arc of a great circle to be drawn through B and D. Then C will be the pole of the arc BD, and the angle C will be measured by BD (Book IX. Prop. VI.), and the angles CBD and D will be right angles. Now before the remaining parts of the quadrantal triangle can be found, at least two parts must be given in addition to the side BC=90°; in which case two parts of the right angled triangle BDA, together with the right angle, become known. Hence the conditions which enable us to determine one of these

"a", "b", "c" on geometric figure